Complexity and Expressivity of Propositional Logics with Team Semantics

ESSLLI 2024 course

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Complexity and Expressivity of Propositional Logics with Team Semantics
Arne Meier, Jonni Virtema
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Lecture 3: Inclusion Logic

Literature: [Hel+19; Hel+20]

Union closure: $T_1 \models \varphi$, $T_2 \models \varphi \Rightarrow T_1 \cup T_2 \models \varphi$

Inspired by „inclusion dependencies“ from database theory.

$$T \models p_1 \cdots p_k \subseteq q_1 \cdots q_k \text{ iff } \forall\, u \in T\ \exists\, v \in T : u(\bar{p}) = v(\bar{q})$$

Lemma 32

$\mathrm{PL}[\subseteq]$ *is union closed.*

Proof by structural induction on φ.

IB $\varphi = p \in \mathrm{PROP}$. $T_1 \models p$ and $T_2 \models p \Rightarrow T_1 \cup T_2 \models p$.

$\varphi = \bar{p} \subseteq \bar{q}$. $T_1 \models \bar{p} \subseteq \bar{q}$ and $T_2 \models \bar{p} \subseteq \bar{q} \Rightarrow \forall u_1 \in T_1\ \exists v_1 \in T_1$ s.t. $u_1(\bar{p}) = v_1(\bar{q})$ and $\forall u_2 \in T_2\ \exists v_2 \in T_2$ s.t. $u_2(\bar{p}) = v_2(\bar{q})$

As a result: $\forall u \in T_1 \cup T_2\ \exists v \in T_1 \cup T_2$ s.t. $u(\bar{p}) = v(\bar{q})$. $\Rightarrow T_1 \cup T_2 \models \bar{p} \subseteq \bar{q}$

IS $\varphi = \alpha \wedge \beta$. By IH, we get that $T_1 \cup T_2 \models \alpha$ and $T_1 \cup T_2 \models \beta$. Hence, $T_1 \cup T_2 \models \alpha \wedge \beta$

$\varphi = \alpha \vee \beta$. By IH, we get that $T_1 \cup T_2 \models \alpha$ and $T_1 \cup T_2 \models \beta$. Hence, $T_1 \cup T_2 \models \alpha \vee \beta$ ∎

Validity in Team Semantics

Classical propositional validity is coNP-complete.

A formula φ is valid if $T \models \varphi$ for all teams T such that the propositions in φ are in the domain of T.

Problem: $\mathrm{VAL}(\mathcal{L})$ – the validity problem for logic $\mathcal{L}$
Input: a $\mathcal{L}$-formula φ
Question: Is φ valid?

Theorem 33

$\mathrm{VAL}(\mathrm{PL}[\subseteq])$ *is* coNP*-complete.*

Hardness follows directly by classical validity which is coNP-complete.

Membership: 1) $PL[\subseteq]$ is union closed, so it suffices to look at singleton teams.

2) Say $\{s\}$ is some singleton team and $\bar{p} \subseteq \bar{q}$ is part of φ.

$\{s\} \models \bar{p} \subseteq \bar{q}$ iff $\bigwedge_{1 \le i \le k} p_i \leftrightarrow q_i$ is satisfied by s

Now, eliminate all inclusion atoms in φ. Call the new formula φ^*.

3) Now, $\varphi \in \mathrm{VAL}(PL[\subseteq])$ iff φ^* is a valid pop. formula.

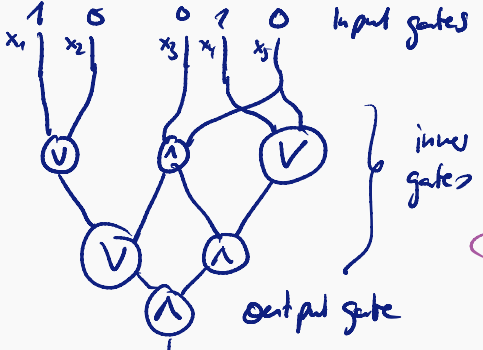
This reduction shows $\mathrm{VAL}(PL[\subseteq]) \in$ coNP. □

Foundations: Monotone circuit value problem

A monotone circuit is a finite directed, acyclic graph in which each node is either:

- an input gate labelled with a Boolean variable x_i,
- a disjunction gate with indegree 2,
- a conjunction gate with indegree 2.

There is exactly one node with outdegree 0, called the output gate.



formula

$$\big((x_1 \vee x_2) \vee (x_3 \wedge x_5)\big) \wedge \big((x_3 \wedge x_5) \wedge (x_4 \vee x_5)\big)$$

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Problem: MCVP — monotone circuit value problem
Input: a monotone circuit C and an input $b_1, \ldots, b_n \in \{0, 1\}$
Question: is the output of the circuit 1

Proposition 34 ([Gol77])

MCVP *is* P*-complete w.r.t.* $\leq_m^{\log}$*-reductions.*

Model-Checking for Inclusion Logic

Theorem 35 ([Hel+19, Thm. 3.5])

$\mathrm{PL}[\subseteq]\text{-}\mathrm{MC}$ *is* P*-complete.*

Model-Checking for Inclusion Logic

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$\mathrm{PL}[\subseteq]$-MC *is* P*-complete.*

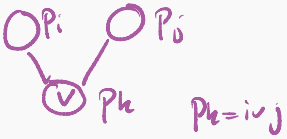
Ideas:

Lower bound: reduce from MCVP

Upper bound: use a labelling algorithm to compute a maximum satisfying team

P-hardness: Idea of the reduction from MCVP to $\mathrm{PL}[\subseteq]$-MC

- gate $g_i \rightsquigarrow$ assignment s_i
- proposition p_i for each gate g_i (where g_0 is the output gate), $p_\bot$ and $p_\top$
- special propositions $p_{k=i\vee j}$ for disjunction gates



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- special propositions $p_{k=i\vee j}$ for disjunction gates
- $s_i \in T$ if g_i has value 1

"correct gate"

$$s_i(p) := \begin{cases} 1 & \text{if } p = p_i \text{ or } p = p_\top, \\ 1 & \text{if } p = p_{k=i\vee j} \text{ or } p = p_{k=j\vee i} \text{ for some } j, k \leq m, \\ 0 & \text{otherwise.} \end{cases}$$

"gate assignment"

P-hardness: Idea of the reduction from MCVP to $\mathrm{PL}[\subseteq]\text{-}\mathrm{MC}$

- gate $g_i \rightsquigarrow$ assignment s_i
- proposition p_i for each gate g_i (where g_0 is the output gate), $p_\bot$ and $p_\top$
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- $s_\bot(p) = 1$ iff $p = p_\bot$ or $p = p_\top$ (no other s_i maps $p_\bot$ to 1)
- create a formula φ_C that quantifies truth value of each gate and ensures correct propagation

More details: P-hardness of $\mathrm{PL}[\subseteq]$-MC.

After skipping some technicalities we arrive at

g_0 is output gate

$$
\begin{array}{rcl}
T \models p_\top \subseteq p_0 & \text{iff} & s_0 \in T \\
T \models p_i \subseteq p_j & \text{iff} & s_i \in T \text{ implies } s_j \in T \\
T \models p_k \subseteq p_{k=i \vee j} & \text{iff} & s_k \in T \text{ implies that } s_i \in T \text{ or } s_j \in T
\end{array}
$$

More details: P-hardness of $\mathrm{PL}[\subseteq]$-MC.

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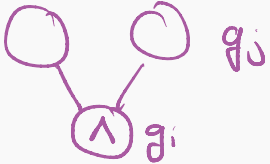
$$\begin{aligned} T \models p_\top \subseteq p_0 \quad &\text{iff} \quad s_0 \in T \\ T \models p_i \subseteq p_j \quad &\text{iff} \quad s_i \in T \text{ implies } s_j \in T \\ T \models p_k \subseteq p_{k=i\vee j} \quad &\text{iff} \quad s_k \in T \text{ implies that } s_i \in T \text{ or } s_j \in T \end{aligned}$$

Recall: gates that are in the team T have a value 1.

Express gate properties:



$$\begin{aligned} \psi_{\mathrm{out}=1} &:= p_\top \subseteq p_0, \\ \psi_\wedge &:= \bigwedge \{ p_i \subseteq p_j \mid (g_j, g_i) \in E \text{ and } \alpha(g_i) = \wedge \}, \\ \psi_\vee &:= \bigwedge \{ p_k \subseteq p_{k=i\vee j} \mid i < j, (g_i, g_k) \in E, (g_j, g_k) \in E, \text{ and } \alpha(g_k) = \vee \} \end{aligned}$$

Encoding MCVP: the final puzzle pieces

More truth about the team:

"input gates with input 1"

$$T := \{s_i \mid \alpha(g_i) \in \{\wedge, \vee\}\} \cup \{s_i \mid \alpha(g_i) \in \{x_i \mid b_i = 1\}\} \cup \{s_\perp\}$$

Encoding MCVP: the final puzzle pieces

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"Wide circuit"

Now we claim that

$$T \models \neg p_\perp \vee (\psi_{\text{out}=1} \wedge \psi_\wedge \wedge \psi_\vee) \quad \text{iff} \quad \text{output of the circuit is 1.}$$

Z Y

$T = Z \cup Y$.

Y : The correct circuit computation and output 1

Z : The "wrong" rest

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Crux 1: split requires guessing a team Y for the right disjunct that encodes the valuation of the circuit.

Encoding MCVP: the final puzzle pieces

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Crux 1: split requires guessing a team Y for the right disjunct that encodes the valuation of the circuit.
Crux 2: $\neg p_\perp$ and $s_\perp$ ensure that Y is nonempty and deal with the propagation of the value 0 by the subformulae of the form $p_i \subseteq p_j$.

Computing a Maximum Satisfying Team

By maxsub(T, φ), we denote the maximum subteam T' of T such that $T' \models \varphi$, i.e., for all $T' \subsetneq T'' \subseteq T$, we have $T'' \not\models \varphi$.
Union closure of $\mathrm{PL}[\subseteq]$ ensures that this always exists.

Lemma 36 (for a proof, see [Hel+19, Lemma 5.1])

If φ is a proposition symbol, its negation, or an inclusion atom, then maxsub(T, φ) can be computed in polynomial time with respect to $|T| + |\varphi|$.

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Interesting case: inclusion atoms (edges between assignments when agree on some variables; successively delete vertices with out-degree 0).

P-algorithm for $\mathrm{PL}[\subseteq]$-MC

Important properties:

- Each team T has a unique maximal subteam satisfying a given formula φ.
- For literals $\mathrm{maxsub}(T, \varphi)$ is computable in polynomial time (Lemma 36).

$\neg p, p, \bar{p} \subseteq \bar{q}$

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Idea of the algorithm checking whether $T \models \varphi$:

1. Build the syntactic tree of φ and label each of its nodes with T.
2. Bottom up part of the algorithm:
   2.1 For literals φ labelled by Y, replace Y by $\mathrm{maxsub}(Y, \varphi)$.
   2.2 For other nodes; update their label depending on their connective, their previous label and their child nodes new labels.
3. Top down part of the algorithm:
   3.1 Starting from root, update labels depending on the connective, previous label and the parent nodes new label.
4. Go to 2.

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The labelling algorithm is decreasing and each round takes only polynomial time.

Membership in P: more formally

approach: use a labelling function f_i of occurrences of subformulae of input φ and start with $f_0(\psi) = T$ for every sub-occurrence ψ

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bottum-up part (odd i):

- for literals ψ: $f_i(\psi) \coloneqq \mathsf{maxsub}(f_{i-1}(\psi), \psi)$
- $f_i(\psi \wedge \theta) \coloneqq f_i(\psi) \cap f_i(\theta)$
- $f_i(\psi \vee \theta) \coloneqq f_i(\psi) \cup f_i(\theta)$

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top-down part (even $i > 0$):

- If $\psi = \theta \wedge \gamma$, let $f_i(\theta) \coloneqq f_i(\gamma) \coloneqq f_i(\theta \wedge \gamma)$.
- If $\psi = \theta \vee \gamma$, let $f_i(\theta) \coloneqq f_{i-1}(\theta) \cap f_i(\theta \vee \gamma)$ and $f_i(\gamma) \coloneqq f_{i-1}(\gamma) \cap f_i(\theta \vee \gamma)$.

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- for literals ψ: $f_i(\psi) := \text{maxsub}(f_{i-1}(\psi), \psi)$
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Claim: $f_\infty(\varphi) = T$ iff $T \models \varphi$

$f_{i+1} \subseteq f_i$

Satisfiability for Inclusion Logic

Theorem 37 ([Hel+20, Cor. 3.6])

$\mathrm{PL}[\subseteq]$-SAT *is* EXP*-complete.*

Short ideas:

Propositional Dynamic Logic

Upper bound: equivalence preserving translation to SAT in PDL with global and converse modalities

Lower bound: reduce from succinct Path-$\mathrm{Systems}$ variant

Satisfiability for Inclusion Logic

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Short ideas:

Upper bound: equivalence preserving translation to SAT in PDL with global and converse modalities

Lower bound: reduce from succinct $\mathrm{Path\text{-}Systems}$ variant (our focus)

L P-complete

Persistent subsets

Definition 38

Let $\mathfrak{A} = (A, S)$ be a structure with $A = \{1, \ldots, n\}$ and $S \subseteq A^3$. A subset P of A is S-persistent if it satisfies the condition

$(*)$ if $i \in P$, then there are $j, k \in P$ such that $(i, j, k) \in S$.

Problem: PER
Input: structures $\mathfrak{A} = (A, S)$ with $A = \{1, \ldots, n\}$ and $S \subseteq A^3$
Question: exists some S-persistent set $P \subseteq A$ such that $n \in P$

Theorem 39 (closely related to $\mathrm{PathSystems}$ [GHR95, p. 171])

PER *is* P*-complete.*

A succinct variant of PER

- represent structures $\mathfrak{A} = (A, S)$ by Boolean circuits C with inputs of length 3ℓ
- $\mathfrak{A} = (A, S) \overset{C}{\rightsquigarrow} (A_C, S_C)$ with $A_C = \{1, \ldots, 2^\ell\}$
- for all $i, j, k \in A$, let $(i, j, k) \in S_C$ if and only if C accepts the input tuple

$$(a_1, \ldots, a_\ell, b_1, \ldots, b_\ell, c_1, \ldots, c_\ell) \in \{0,1\}^{3\ell},$$

where $i = \mathrm{bin}(a_1 \ldots a_\ell)$, $j = \mathrm{bin}(b_1 \ldots b_\ell)$ and $k = \mathrm{bin}(c_1 \ldots c_\ell)$.

Problem: S-PER
Input: Boolean circuits C with inputs of length 3ℓ , $\mathfrak{A} = (A, S)$
Question: exists some S_C-persistent set $P \subseteq A_C$ such that $2^\ell \in P$

Theorem 40 ([Hel+19, Lem. 3.4])

S-PER *is* EXP*-hard with respect to* $\leq_m^p$*-reductions.*

EXP-**hardness of** $\mathrm{PL}[\subseteq]$**: prerequisites**

To show: $\text{S-PER} \leq_m^p \mathrm{PL}[\subseteq]\text{-SAT}$.

Notation: $T(p_1, \ldots, p_n) \coloneqq \{(s(p_1), \ldots, s(p_n)) \in \{0,1\}^n \mid s \in T\}$

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Notation: $T(p_1, \ldots, p_n) \coloneqq \{(s(p_1), \ldots, s(p_n)) \in \{0,1\}^n \mid s \in T\}$

Note that the semantics of inclusion atoms can now be expressed as

$$T \models p_1 \cdots p_n \subseteq q_1 \cdots q_n \iff T(p_1, \ldots, p_n) \subseteq T(q_1, \ldots, q_n).$$

Encoding S-PER into PL[⊆]

C is a Boolean circuit with 3ℓ input gates ordered $g_1, \ldots, g_m$ such that $g_1, \ldots, g_{3\ell}$ are the input gates and g_m is the output gate.

- fix propositions p_i for each gate g_i
- define for each gate a $\mathrm{PL}[\subseteq]$ formula θ_i:

$$\theta_i = \begin{cases} p_i \leftrightarrow \neg p_j & \text{if } g_i \text{ is a NOT gate with input } g_j \\ p_i \leftrightarrow (p_j \wedge p_k) & \text{if } g_i \text{ is an AND gate with inputs } g_j \text{ and } g_k \\ p_i \leftrightarrow (p_j \vee p_k) & \text{if } g_i \text{ is an OR gate with inputs } g_j \text{ and } g_k \end{cases}$$

Note: $\leftrightarrow$ is usual shorthand for flat formulas

↳ "$\vee$" coincides with classical "or"-semantics

Encoding S-PER into PL[⊆]

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Note: $\leftrightarrow$ is usual shorthand for flat formulas

Then: $\psi_C := \left(\bigwedge_{3\ell+1 \le i \le m} \theta_i\right) \wedge p_m$. (truth values of p_i match acc. computation of C)

"inner gates" "output is 1"

Encoding persistency into the formula

Input gate propositions: $p_1, \ldots, p_\ell, \underbrace{p_{\ell+1}, \ldots, p_{2\ell}}_{=:q_1,\ldots,q_\ell}, \underbrace{p_{2\ell+1}, \ldots, p_{3\ell}}_{=:r_1,\ldots,r_\ell}$

Encoding persistency into the formula

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"Correct acc. comp"

$$\varphi_C := \psi_C \wedge q_1 \cdots q_\ell \subseteq p_1 \cdots p_\ell \wedge r_1 \cdots r_\ell \subseteq p_1 \cdots p_\ell \wedge p_m \cdots p_m \subseteq p_1 \cdots p_\ell$$

Claim: C is a positive instance of $\mathrm{S\text{-}PER}$ if and only if φ_C is satisfiable.

Encoding persistency into the formula

Input gate propositions: $p_1, \ldots, p_\ell, \underbrace{p_{\ell+1}, \ldots, p_{2\ell}}_{=:q_1,\ldots,q_\ell}, \underbrace{p_{2\ell+1}, \ldots, p_{3\ell}}_{=:r_1,\ldots,r_\ell}$ The final formula:

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Claim: C is a positive instance of $\mathrm{S\text{-}PER}$ if and only if φ_C is satisfiable.

We will prove only $\Rightarrow$. For the other direction, we define the S_C-persistent set as

$$\{\mathrm{bin}(a_1 \ldots a_\ell) \mid (a_1, \ldots, a_\ell) \in T(p_1, \ldots, p_\ell)\}.$$

"⇒" of the claim

C is a pos. instance for S-PER.
$\Rightarrow$ there is a S_C-persistent set $P \subseteq A_C$ s.t. $2^\ell \in P$ (*)
Let T be a tree s.t. $(a_1, \dots, a_m) \in T$ iff $bin(a_1 \dots a_m), bin(a_{\ell+1} \dots a_{2\ell}), bin(a_{2\ell+1} \dots a_{3\ell}) \in P$.

To show: $T \models \varphi_C$.

1) $T \models \varphi_C$ ✓ as we defined T to suffice persistency.
2) $T \models q_1 \dots q_\ell \subseteq p_1 \dots p_\ell$. For that assume $(b_1, \dots, b_\ell) \in T(q_1, \dots, q_\ell)$
   Then: $i := bin(b_1 \dots b_\ell) \in P$ and $j, k \in P$ s.t. $(i,j,k) \in S_C$ true due to (*)
   Thus: we have $(a_1, \dots, a_m)$ corresp. to an accepting computation s.t.
   $(a_1, \dots, a_\ell) = (b_1, \dots, b_\ell)$, $j = bin(b_{\ell+1} \dots b_{2\ell})$, $k = bin(a_{2\ell+1} \dots a_{3\ell})$
   $\Rightarrow (a_1, \dots, a_m) \in T$ and $(b_1, \dots, b_\ell) \in T(p_1, \dots, p_\ell)$

3) Similarly for $T \models r_1 \cdots r_\ell \subseteq p_1 \cdots p_\ell$

4) Since $T \models p_m \Rightarrow T(p_m, \dots, p_m) = \{(1, \dots, 1)\}$

Since $2^\ell = bin(1, \dots, 1) \in P$ there is $(a_1, \dots, a_m) \in T$ s.t.

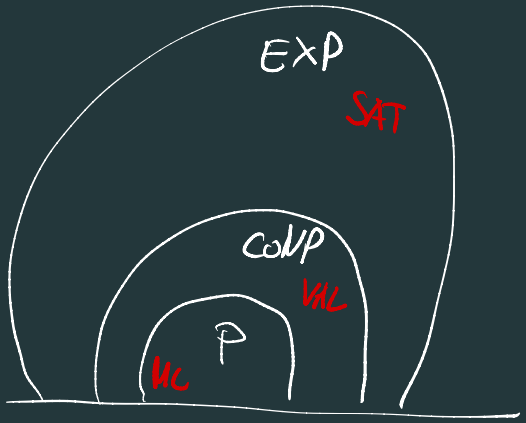
$(a_1, \dots, a_\ell) = (1, \dots, 1)$.

Thus $(1, \dots, 1) \in T(p_1, \dots, p_\ell)$ and $T \models p_m \cdots p_m \subseteq p_1 \cdots p_\ell$.

□

Conclusion of Lecture 3

- $\mathrm{PL}[\subseteq]$-MC is P-complete.
- $\mathrm{PL}[\subseteq]$-SAT is EXP-complete.
- $\mathrm{PL}[\subseteq]$-VAL is coNP-complete.



Complexity and Expressivity of Propositional Logics with Team Semantics
Arne Meier, Jonni Virtema
8th of August

Lecture 4: Complexity of propositional dependence logic and beyond

Literature: [Vir17; Han+18]

Bibliography i

[BRV01] Patrick Blackburn, Maarten de Rijke and Yde Venema. *Modal Logic*. Cambridge Tracts in Theoretical Computer Science. Cambridge University Press, 2001. DOI: 10.1017/CBO9781107050884.

[CES86] E. Clarke, E. Allen Emerson and A. Sistla. 'Automatic Verification of Finite-State Concurrent Systems Using Temporal Logic Specifications'. In: *ACM Transactions on Programming Languages and Systems* 8.2 (1986), pp. 244–263.

[Coo71] Stephen A. Cook. 'The Complexity of Theorem-Proving Procedures'. In: *Proceedings of the 3rd Annual ACM Symposium on Theory of Computing, May 3-5, 1971, Shaker Heights, Ohio, USA*. Ed. by Michael A. Harrison, Ranan B. Banerji and Jeffrey D. Ullman. ACM, 1971, pp. 151–158. DOI: 10.1145/800157.805047. URL: https://doi.org/10.1145/800157.805047.

Bibliography ii

[EFT94] Heinz-Dieter Ebbinghaus, Jörg Flum and Wolfgang Thomas. *Mathematical logic (2. ed.)* Undergraduate texts in mathematics. Springer, 1994.

[EJ99] E. Allen Emerson and Charanjit S. Jutla. 'The Complexity of Tree Automata and Logics of Programs'. In: *SIAM J. Comput.* 29.1 (1999), pp. 132–158.

[ES84] E. Allen Emerson and A. Prasad Sistla. 'Deciding Full Branching Time Logic'. In: *Inf. Control.* 61.3 (1984), pp. 175–201.

[FL79] Michael J. Fischer and Richard E. Ladner. 'Propositional Dynamic Logic of Regular Programs'. In: *J. Comput. Syst. Sci.* 18.2 (1979), pp. 194–211.

[GHR95] Raymond Greenlaw, H. James Hoover and Walter L. Ruzzo. *Limits to Parallel Computation: P-completeness Theory*. New York, NY, USA: Oxford University Press, Inc., 1995. ISBN: 0-19-508591-4.

Bibliography iii

[Gol77] L. M. Goldschlager. 'The monotone and planar circuit value problems are log-space complete for P'. In: *SIGACT News* 9 (1977), pp. 25–29.

[Gut+22] Jens Oliver Gutsfeld, Arne Meier, Christoph Ohrem and Jonni Virtema. 'Temporal Team Semantics Revisited'. In: *LICS '22: 37th Annual ACM/IEEE Symposium on Logic in Computer Science, Haifa, Israel, August 2 - 5, 2022*. Ed. by Christel Baier and Dana Fisman. ACM, 2022, 44:1–44:13. DOI: 10.1145/3531130.3533360. URL: https://doi.org/10.1145/3531130.3533360.

[Han+18] Miika Hannula, Juha Kontinen, Jonni Virtema and Heribert Vollmer. 'Complexity of Propositional Logics in Team Semantic'. In: *ACM Trans. Comput. Log.* 19.1 (2018), 2:1–2:14.

Bibliography iv

[Han19] Miika Hannula. 'Validity and Entailment in Modal and Propositional Dependence Logics'. In: *Logical Methods in Computer Science* Volume 15, Issue 2 (Apr. 2019). DOI: 10.23638/LMCS-15(2:4)2019. URL: https://lmcs.episciences.org/5403.

[Hel+14] Lauri Hella, Kerkko Luosto, Katsuhiko Sano and Jonni Virtema. 'The Expressive Power of Modal Dependence Logic'. In: *Advances in Modal Logic*. College Publications, 2014, pp. 294–312.

[Hel+19] Lauri Hella, Antti Kuusisto, Arne Meier and Jonni Virtema. 'Model checking and validity in propositional and modal inclusion logics'. In: *J. Log. Comput.* 29.5 (2019), pp. 605–630.

[Hel+20] Lauri Hella, Antti Kuusisto, Arne Meier and Heribert Vollmer. 'Satisfiability of Modal Inclusion Logic: Lax and Strict Semantics'. In: *ACM Trans. Comput. Log.* 21.1 (2020), 7:1–7:18.

Bibliography v

[HS15] Lauri Hella and Johanna Stumpf. 'The expressive power of modal logic with inclusion atoms'. In: *GandALF*. Vol. 193. EPTCS. 2015, pp. 129–143.

[KV85] Gabriel M. Kuper and Moshe Y. Vardi. 'On the Expressive Power of the Logical Data Model (Preliminary Report)'. In: *SIGMOD Conference*. ACM Press, 1985, pp. 180–187.

[Lev73] Leonid A. Levin. 'Universal sequential search problems'. In: *Problemy Peredachi Informatsii* 9.3 (1973).

[Loh12] Peter Lohmann. 'Computational Aspects of Dependence Logic'. PhD thesis. Leibniz Universität Hannover, 2012. arXiv: 1206.4564. URL: http://arxiv.org/abs/1206.4564.

[LV19] Martin Lück and Miikka Vilander. 'On the Succinctness of Atoms of Dependency'. In: *Log. Methods Comput. Sci.* 15.3 (2019).

Bibliography vi

[Pap07] Christos H. Papadimitriou. *Computational complexity*. Academic Internet Publ., 2007.

[Pra80] V. R. Pratt. 'A near-optimal method for reasoning about action'. In: *Journal of Computer and System Sciences* 20.2 (1980), pp. 231–254.

[SC85] A. Prasad Sistla and Edmund M. Clarke. 'The Complexity of Propositional Linear Temporal Logics'. In: *J. ACM* 32.3 (1985), pp. 733–749.

[Sch02] P. Schnoebelen. 'The Complexity of Temporal Logic Model Checking'. In: *Advances in Modal Logic*. Vol. 4. 2002, pp. 393–436.

[Sip97] Michael Sipser. *Introduction to the theory of computation*. PWS Publishing Company, 1997.

[Var09] Moshe Y. Vardi. 'From Philosophical to Industrial Logics'. In: *ICLA*. Vol. 5378. Lecture Notes in Computer Science. Springer, 2009, pp. 89–115.

Bibliography vii

[Vir+21] Jonni Virtema, Jana Hofmann, Bernd Finkbeiner, Juha Kontinen and Fan Yang. 'Linear-Time Temporal Logic with Team Semantics: Expressivity and Complexity'. In: *FSTTCS*. Vol. 213. LIPIcs. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2021, 52:1–52:17.

[Vir17] Jonni Virtema. 'Complexity of validity for propositional dependence logics'. In: *Inf. Comput.* 253 (2017), pp. 224–236.

[YV17] Fan Yang and Jouko Väänänen. 'Propositional team logics'. In: *Ann. Pure Appl. Log.* 168.7 (2017), pp. 1406–1441. DOI: 10.1016/J.APAL.2017.01.007. URL: https://doi.org/10.1016/j.apal.2017.01.007.