Advanced Logics | Institut für Theoretische Informatik
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Exercise sheet 9

Exercise 1: State an SO(∃) formula $\psi_{3\mathrm{SAT}}$ which encodes satisfiability of propositional formulas.

Definition. An *alternating graph* $G = (V, E, A, s, t)$ is a directed graph with *existential* and *universal* vertices. $A \subseteq V$ is a the set of universal vertices. Let $\tau_{ag} = (E^2, A^1, s, t)$ be the vocabulary for alternating graphs.
For alternating graphs there exists a new meaning of reachability. Let $P_a^G(x, y)$ be the smallest relation on vertices of G with the following properties:

1. $P_a^G(x, x)$ holds
2. If x is existential and $P_a^G(z, y)$ holds for edge (x, z) then $P_a^G(x, y)$ is also true.
3. If x is universal then there exists at least one edge leaving x. Further if $P_a^G(z, y)$ holds for all edges (x, z) then $P_a^G(x, y)$ is true.

Formally let ALTGAP $:= \{(G, s, t) \mid P_a^G(s, t)\}$.
The notion of alternating Turing machines is known from the lecture.

Exercise 2: Prove the following. ALTGAP is complete for P under first-order reductions.

Hint: you may use the fact that P *coincides with the set of languages which can be accepted by alternating LOGSPACE Turing machines.*

Exercise 3: In the proof of Fagin's Theorem we use a specific ordering relation on k-ary elements. Construct an SO(∃) formula $\phi \equiv \exists S^{2k}\psi$ such that there exists a $2k$-ary predicate $S^{2k}(\overline{x}, \overline{y})$ which holds iff $\overline{x} < \overline{y}$ holds.