Preface

This introductory textbook presents an algorithmic and computability based approach to circuit complexity. Intertwined with the consideration of practical examples and the design of efficient circuits for these, a lot of care is spent on the formal development of the computation model of uniform circuit families and the motivation of the complexity classes defined in this model.

Boolean circuits gain much of their attractiveness from the fact that nontrivial lower bounds for the complexity of particular practical problems in this model are known. However, I feel that circuit complexity should not be reduced to the theory of lower bounds. In the past few years, the study of complexity classes defined by Boolean circuits, as well as their properties and how they relate to other computational devices, has become more and more important. Best known are probably the classes that form the NC hierarchy, which was defined to capture the notion of problems that have very fast parallel algorithms using a feasible amount of hardware. This study is thus motivated from an algorithmic point of view, and therefore an important issue is the so-called *uniformity* of circuits, because only a uniform circuit family can be regarded as an implementation of an algorithm.

This book presents some classical as well as some recent lower bound proofs; however, I restrict myself to a small and subjective selection of important examples. The reader who is interested in exploring this topic further is directed to other textbooks (an extensive list of pointers to excellent literature is given in the Bibliographic Remarks of this book). Here, instead, the theory of circuit complexity classes, mostly connected with the NC hierarchy mentioned above, is developed thoroughly.

It is difficult, in times of such rapid developments in a field as we witness today in circuit complexity, to write a textbook which is not already out of date when it is published. Therefore the material presented here is chosen from a basic core of fundamental results which underlies most current research in circuit complexity and hopefully will still be important in the future. Of course a textbook of this size cannot even touch all important and interesting developments in circuit complexity. However, I have increased the number of topics and results by covering some of them in the exercises, and introducing others in the Bibliographic Remarks sections.

The first three chapters can be used for a one-semester introduction to circuit complexity on the advanced undergraduate or first-year graduate level. (For readers familiar with the German university system, I have used Chaps. 1–3 for a Hauptstudiumsvorlesung Schaltkreiskomplexität for students with a moderate background in theoretical computer science.) The second half of the book, suited for an ongoing course or an advanced seminar, requires more experience from the reader, but is certainly still accessible to first-year graduate students.

The reader is expected to possess some knowledge and experience about formal languages and complexity of deterministic and nondeterministic Turing machines, as usually gained in a one-semester introduction to foundations of computing (refer, e. g., to [HU79]). Besides this only basic mathematics is required (however, experience in a standard programming language will be helpful). An exception from this is Chap. 6. While I have tried to keep the exposition self-contained by introducing all necessary concepts, a full comprehension of most of the results presented in this final chapter will probably only be possible for a student familiar with the basic notions and results from complexity theory.

A few notes about how to use this book: Generally, each chapter presupposes the material of the preceding chapters. It is suggested that the reader quickly browse the Appendix "Mathematical Preliminaries" (p. 233) before starting with the main text, and then later study them in more detail as necessary. Each chapter ends with a number of exercises. Most exercises can be answered immediately if the material of the text has been understood. Starred exercises require some afterthought and more time. Double-starred exercises present quite difficult or lengthy problems. For most starred and all double-starred exercises, a pointer to the literature where a solution can be found is given.

Current information on this book may be found on the Web at the following location:

`http://www-info4.informatik.uni-wuerzburg.de/CC/`

If you find errors in the book, please report them using the above URL; or send an e-mail to vollmer@informatik.uni-wuerzburg.de.

Many people have contributed to the development of this book with helpful hints, discussions, and proof reading of the manuscript. I am grateful to Eric Allender, David Mix Barrington, Christian Glaßer, Georg Gottlob, Sven Kosub, Pierre McKenzie, Ken Regan, Steffen Reith, Heinz Schmitz, Klaus W. Wagner, and, in particular, to Ulrich Hertrampf. I also thank Hans Wössner and Ingeborg Mayer from Springer-Verlag for their excellent cooperation, and the copy editor, Reginald Harris, for many very helpful remarks.

Würzburg, Spring 1999 *Heribert Vollmer*