A Short Overview of Hinges

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History

Context (1982)

- Dependency theory
- Hypergraphs

Original goal

Decomposing a join dependency into a set of join dependencies with fewer components than the original one.

Join dependencies

Definition

Let R be a set of attributes, and let $X_1, \ldots, X_n \subset R$ with $\bigcup_{i=1}^{n} X_i = R$.

A relation r over R satisfies the join dependency

$$X_1 \bowtie \cdots \bowtie X_n$$

if

$$\pi_{X_1}(r) \bowtie \cdots \bowtie \pi_{X_n}(r) = r.$$

Hypergraph representation

The join dependency $X_1 \bowtie \cdots \bowtie X_n$ can be represented as the hypergraph

$$\mathcal{H} = (R, \{X_1, \ldots, X_n\}).$$

Hinge

Preceding remark

Hinges are an edge-based notion.

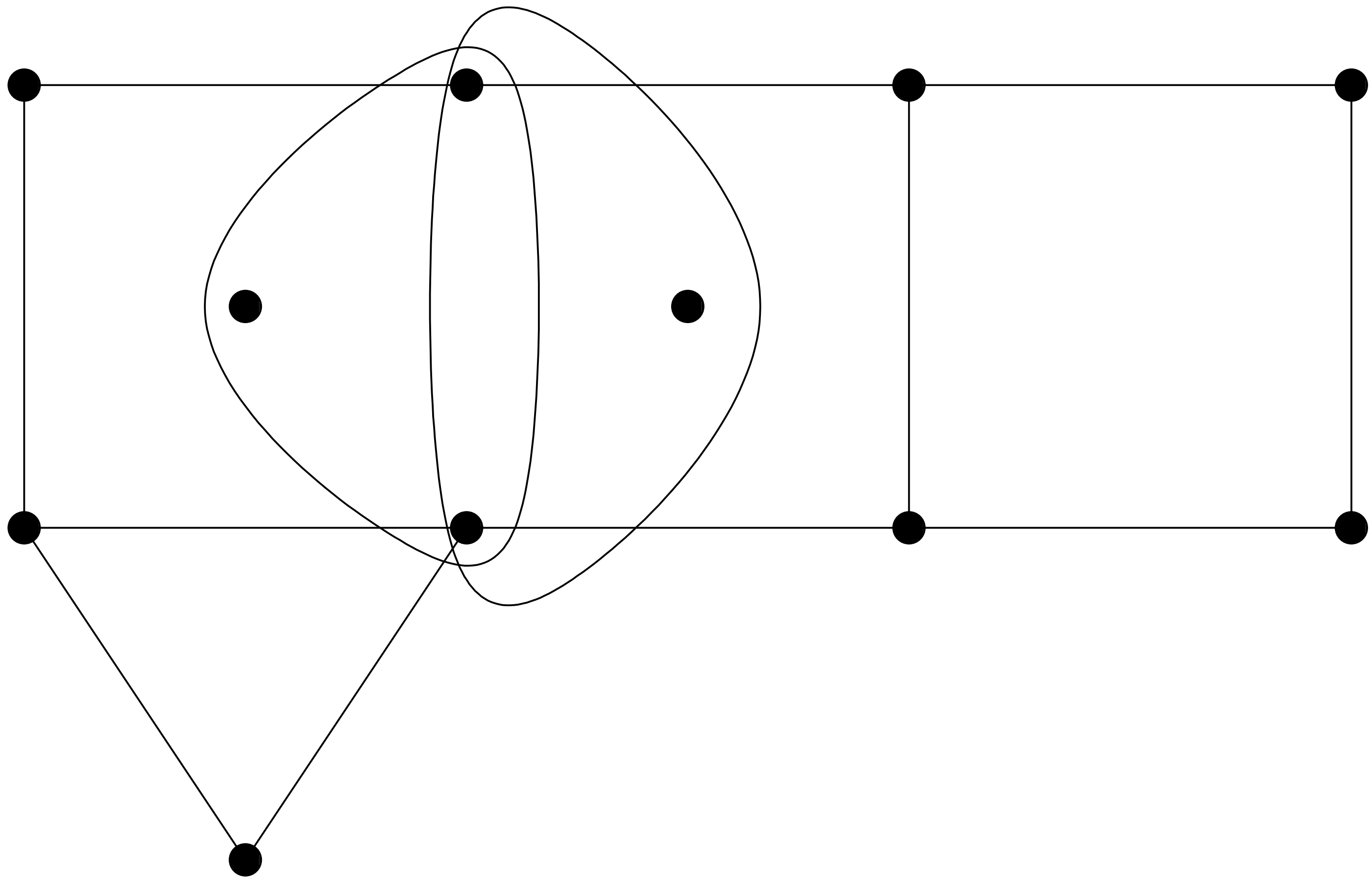
Relative connectedness

- For a reduced hypergraph $\mathcal{H} = (V, E)$ and a subset H of E, edges e and f of E are connected w.r.t. H if e and f do not meet entirely inside $\bigcup H$.
- Connected components of E w.r.t. H.

Definition of a hinge

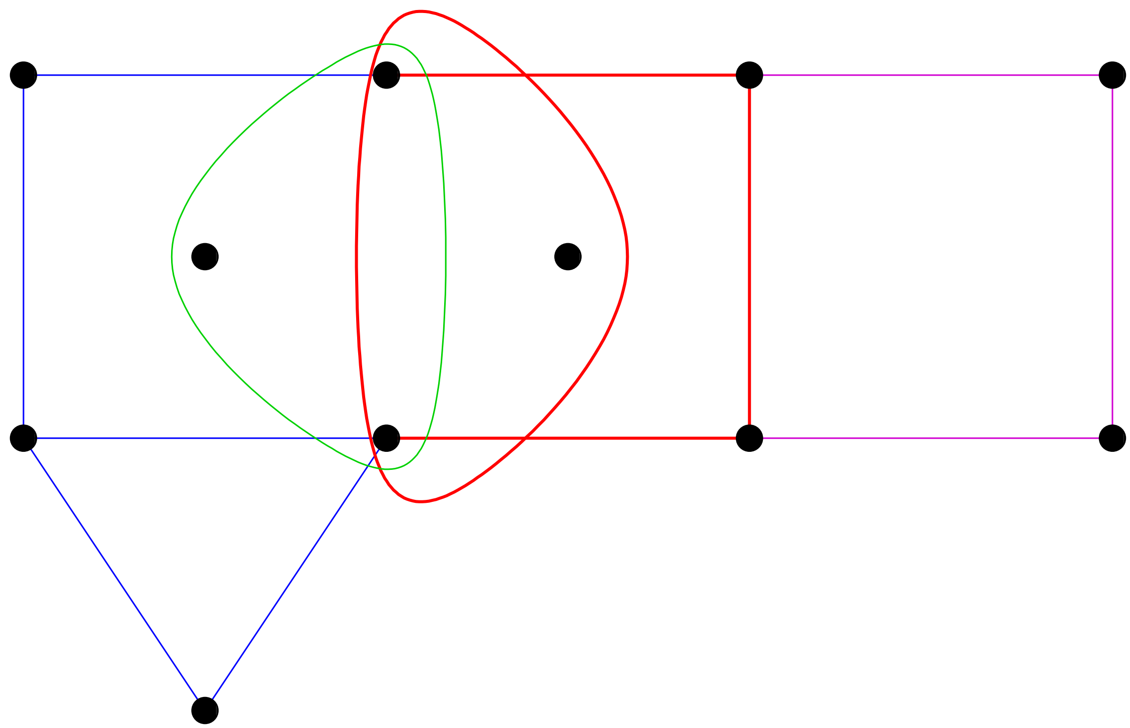
For a reduced hypergraph $\mathcal{H} = (V, E)$, a subset H of E is a hinge if, for each connected component $F \subseteq E$ w.r.t. H, $\bigcup F$ meets $\bigcup H$ within a single edge of H (called a separating edge for that component).

Running example



Hinges (continued)

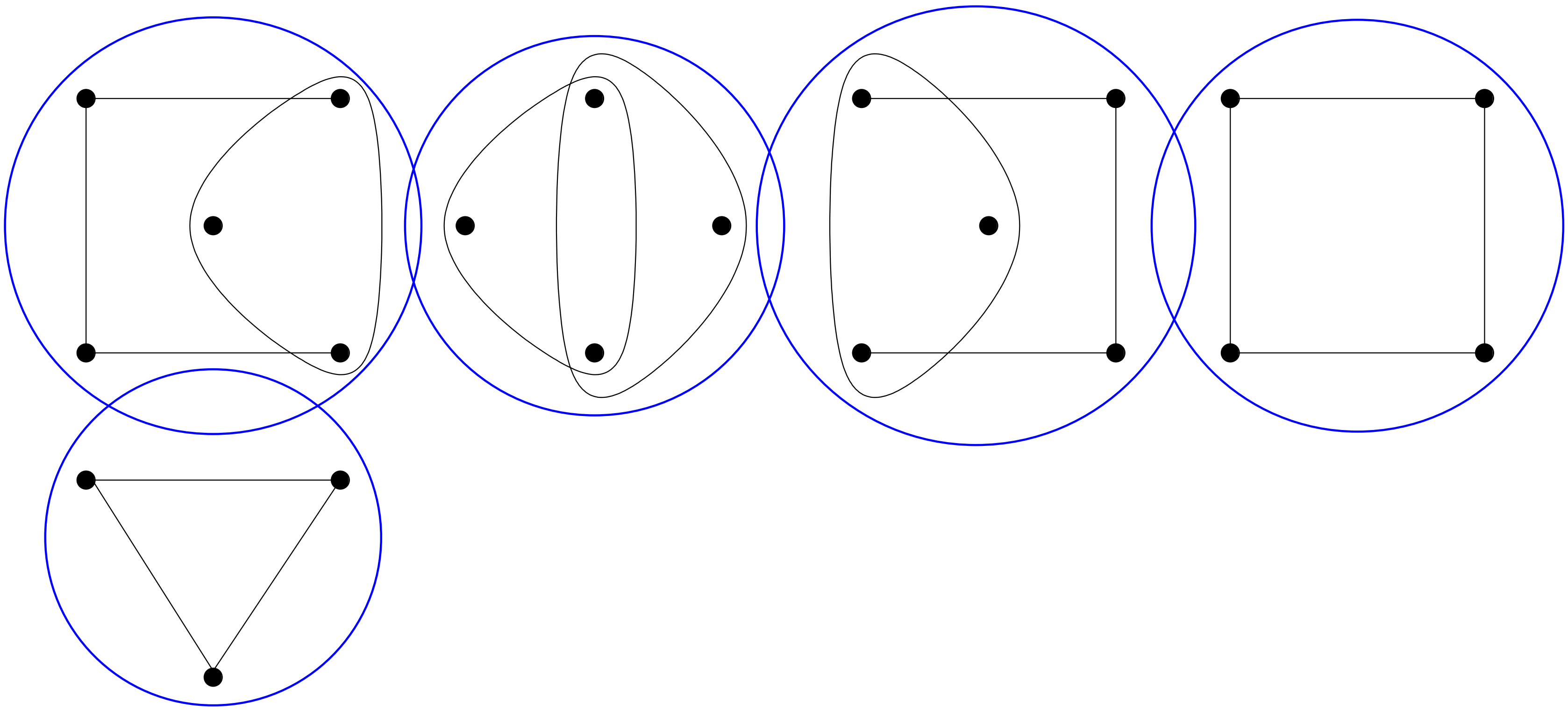
- Example of a hinge.



- Arbitrary hinges vs. minimal hinges.

Hinge tree

- Join tree of minimal hinges.
- Example of a hinge tree.

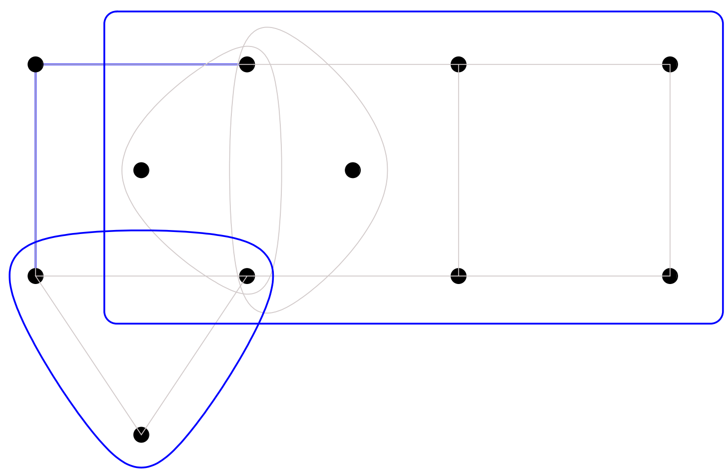


- Notice that not all minimal hinges have been used.

Hinge trees and join dependencies

Associating a join dependency to a hinge

- Add the connected components with respect to the hinge to their corresponding separating edges.
- Example of associating a join dependency to a hinge.



Decomposition result

The original join dependency is equivalent to the set of join dependencies associated to the minimal hinges of a hinge tree.

Results in the database context

Degree of cyclicity

- Hinge trees can be computed in uniform polynomial time.
- The size (in number of edges) of the largest hinge in any hinge tree is an invariant, called the degree of cyclicity, and is equal to the size of the largest minimal hinge in the hypergraph.
- The degree of cyclicity can be computed in polynomial time.
- A hypergraph is acyclic if and only if its degree of cyclicity is 2.

Partial consistency and global consistency

Definitions

- Let $\mathcal{H} = (R, \{X_1, \ldots, X_n\})$ be a hypergraph with $R = \bigcup_{i=1}^{n} X_i$, and let $r_1, \ldots, r_n$ be relations over $X_1, \ldots, X_n$. Then, $r_1, \ldots, r_n$ are k-wise consistent if, for all $I \subseteq \{1, \ldots, n\}$ with $|I| = k$, and for all i in I,
$$\pi_{X_i}(\bowtie_{\ell \in I} r_\ell) = r_i.$$
- Global consistency is n-wise consistency.

Remark

Notice the difference with the similar notion in Scarcello's talk!

Result from the theory of acyclicity

A hypergraph is acyclic if and only if pairwise consistency implies global consistency.

Results in the database context (continued)

Relationship between partial and global consistency

- A hypergraph has degree of cyclicity at most k if and only if k-wise consistency implies global consistency.
- Generalization of the characterization of acyclicity on the previous slide.

Results in the CSP-context

Characterization of a hinge

For a minimal set of constraints, a partial solution to a subset of the scopes can always be extended to a global solution if and only if that subset is a hinge (under moderate conditions on the size of the domains).

Hybrid algorithm

The tree clustering algorithm can be improved by first finding a hinge tree and then applying tree clustering to each hinge individually.

Theory of decompositions

Situation of hinge trees and hinge trees + tree clustering in the theory of decompositions

See the talks of Scarcello and Cohen.