Constraint Satisfaction with Succinctly Specified Relations

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Joint work with Hubie Chen

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- $\mathcal{B}(\mathcal{I})$ structure induced by constraint relations on the domain.
  Restrictions on $\mathcal{B}(\mathcal{I})$ are called constraint language restrictions.

Uniform CSPs

For classes $\mathbf{K}, \mathbf{L}$ of relational structures:

CSP($\mathbf{K}, \mathbf{L}$)

Instance: CSP instance $\mathcal{I}$
with $\mathcal{A}(\mathcal{I}) \in \mathbf{K}$ and $\mathcal{B}(\mathcal{I}) \in \mathbf{L}$.

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- We write CSP(**K**, $-$) or CSP($-$, **L**) if **L** resp. **K** is the class of all structures.
- We are mainly interested in restrictions of the form CSP(**K**, $-$) (structural restrictions).

Tractable structural restrictions

Two important ideas

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Theorem (Dalmau, Kolaitis, and Vardi 2002)

Let **K** *be a class of structures of bounded tree width modulo homomorphic equivalence.*
Then $\mathrm{CSP}(\mathbf{K}, -)$ *is tractable.*

Bounded arity

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Assume that FPT $\neq$ W[1] *(or stronger:* 3-SAT $\notin$ DTIME$(2^{o(n)})$*).*
For every recursively enumerable class **K** *of structures of bounded arity, the following are equivalent:*

1. CSP$(\mathbf{K}, -)$ *is tractable.*
2. **K** *has bounded tree width modulo homomorphic equivalence.*

Digression — a few remarks on the theorem

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  There are classes **K** of graphs of unbounded treewidth modulo homomorphic equivalence such that CSP(**K**, $-$) $\in$ NP[$\log^2 n$].

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  If FPT $=$ W[1] then there are classes **K** of graphs of unbounded treewidth modulo homomorphic equivalence such that CSP(**K**, $-$) is tractable.

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- This is not a dichotomy theorem.
- The assumption FPT $\neq$ W[1] cannot be replaced by anything weaker.
- The requirement that **K** be recursively enumerable is inessential. With a slightly stronger complexity theoretic assumption, we can get rid of it.

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A new version of the paper is on my webpage:

http://www.informatik.hu-berlin.de/~grohe/pub/gro06b+.html

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Let k be a constant and $\mathcal{I} = (V, D, C)$ a CSP-instance. Suppose that the constraints

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cover all variables, i.e., $V \subseteq \{x_{ij} \mid 1 \leq i \leq k, 1 \leq j \leq r_i\}$.

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Then for the set S of all solutions of $\mathcal{I}$, we have:

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($S \subseteq R_1 \times \cdots \times R_k$ if the variable sets are disjoint). Hence

$$|S| \leq |R_1| \cdot |R_2| \cdots |R_k| \leq ||\mathcal{I}||^k,$$

and S can be computed in polynomial time.

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That's all great — but wait . . .

...haven't we just proved $(N = 1 \vee P = 0)$

- If viewed as a (Boolean) CSP, the satisfiability problem SAT(Φ) for the following class of formulas is tractable:

$$\Phi := \{\phi \wedge (x_1 \vee \ldots \vee x_n) \mid n \geq 1, \phi \text{ CNF formula with variables among } x_1, \ldots, x_n\}$$

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- Hence P = NP.

Conclusion

- By a simple reduction to constraint satisfaction problems, we have proved the satisfiability problem to be in polynomial time and hence $\mathrm{P} = \mathrm{NP}$.
- This impressively shows the power of the algorithmic ideas developed in the area.

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In this work, we try to initiate a systematic study of the complexity of CSP with implicitly represented relations.

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DNF-formulas form natural large class of formulas with a tractable satisfiability problem.

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DNF Representations

DNF-formulas form natural large class of formulas with a tractable satisfiability problem.
Therefore, we decided to choose DNF-formulas as succinct representations of Boolean constraint relations.

Arbitrary domains — the GDNF-representation

Definition

Let D be a set. A generalised DNF (GDNF) representation of a relation $R \subseteq D^k$ is an expression of the form

$$\bigcup_{i=1}^{m} (P_{i1} \times \cdots \times P_{ik})$$

where $m \geq 0$ and $P_{ij} \subseteq D$ for $1 \leq i \leq m, 1 \leq j \leq k$.

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Remark

The GDNF enables us to represent relations of size $\Omega(D^k)$ by expressions of size $O(m \cdot |D| \cdot k)$.

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Let MAX $\subseteq [n]^{k+1}$ be defined by

$$\text{MAX} = \big\{(x_1, \ldots, x_k, y) \bigm| y = \max\{x_1, \ldots, x_k\}\big\}$$

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Then $|\text{MAX}| = \Omega(n^k)$.

MAX has the following GDNF-representation of size $O(k^2 \cdot n^2)$:

$$\bigcup_{\substack{1 \le i \le k \\ 1 \le j \le n}} \Big([j] \times \ldots \times [j] \times \underbrace{\{j\}}_{i\text{th place}} \times [j] \times \ldots \times [j] \times \underbrace{\{j\}}_{(k+1)\text{st place}} \Big).$$

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Definition

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- A CSP-instance $\mathcal{I}$ is represented in GDNF if all constraint relations are represented in GDNF.
- By $\mathrm{CSP}_{\mathrm{GDNF}}(\mathbf{K}, \mathbf{L})$ is the version of the problem $\mathrm{CSP}(\mathbf{K}, \mathbf{L})$ where instances are represented in GDNF.

Definition

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$$A_I = A \cup \bigcup_{R \in \sigma} \{(R, a_1, \ldots, a_k) : (a_1, \ldots, a_k) \in R^{\mathcal{A}}\},$$

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   - relations

$$R_i^{\mathcal{A}_I} = \{((R, a_1, \ldots, a_k), a_i) : (a_1, \ldots, a_k) \in R^{\mathcal{A}}\}$$

   for all k-ary $R \in \tau$ and $i \in [k]$.

Example

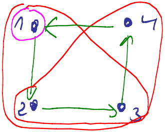
$\{R_3, E_2, P_1\}$ - structure $\mathcal{A}$:

$A = \{1, 2, 3, 4\}$

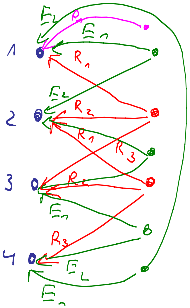
$R^{\mathcal{A}} = \{(1,2,3), (2,3,4)\}$

$E^{\mathcal{A}} = \{(1,2), (2,3), (3,4), (4,1)\}$

$P^{\mathcal{A}} = \{(1)\}$



The structure $\mathcal{A}_I$



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The incidence width $\text{iw}(\mathcal{A})$ of a relational structure $\mathcal{A}$ is the tree width of its incidence structure.

Incidence width

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Remark
For all structures $\mathcal{A}$ of arity k:

$$\mathrm{iw}(\mathcal{A}) - 1 \leq \mathrm{tw}(\mathcal{A}) \leq (\mathrm{iw}(\mathcal{A}) + 1) \cdot (k - 1).$$

Theorem

Assume that FPT $\neq$ W[1].

For every recursively enumerable class **K** *of structures of bounded arity, the following are equivalent:*

1. $\mathrm{CSP}_{\mathrm{GDNF}}(\mathbf{K}, -)$ *is tractable.*
2. **K** *has bounded incidence width modulo homomorphic equivalence.*

Constraint language restrictions

Theorem

Let $\mathbf{L}_k$ *be the class of all relational structures having a near-unanimity polymorphism of arity* k*. For each* $k \geq 3$*, the problem* $\mathrm{CSP}_{\mathrm{GDNF}}(-, \mathbf{L}_k)$ *is tractable.*

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Theorem

Let $\mathbf{L}_k$ be the class of all relational structures having a near-unanimity polymorphism of arity k. For each $k \geq 3$, the problem $\mathrm{CSP}_{\mathrm{GDNF}}(-, \mathbf{L}_k)$ is tractable.

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Let $\mathbf{L}$ be the set of all relational structures invariant under a set function. The problem $\mathrm{CSP}_{\mathrm{GDNF}}(-, \mathbf{L})$ is tractable.

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Other representations

- Ordered binary decision diagrams (OBDDs) are another representation of Boolean relations with nice algorithmic properties. In particular, the satisfiability problem is decidable.
- We considered the natural generalization of OBDDs to arbitrary finite domains and obtained initial results.